Correlation Between Visual Magnitudes and the Outgassing Rate of CO in Comets Beyond 3 AU*

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Abstract. Since its first detection at radio wavelengths in comet 29P/Schwassmann-Wachmann in 1993, the outgassing of CO has been observed as the likely mechanism to sustain activity in comets beyond 3 AU from the sun. A sensitive search for CO in five comets showing sustained activity beyond that distance has been conducted. The comparison between the outgassing rates of CO (Q_{CO}) *and the heliocentric magnitudes* ($H_\Delta = m_1 - 5 \log \Delta$) *of these comets shows a strong linear correlation for the two long-period comets well detected in CO, namely C/1995 O1 (Hale-Bopp) and C/1997 J2 (Meunier-Dupouy). The inferred correlation formula,* $\log(Q_{CO}) = 30.0 - 0.29\, H_\Delta$, *is not followed by 29P, which is very different and/or belongs to another distinct group of dust-poor comets that may have a larger outgassing rate of CO for a given magnitude* H_Δ. *The slope of the formula, "−0.29", is likely to be less comet-dependent and might be used to infer activity indexes (n) for distant comets. Given the current performances of the submillimeter-wavelength radio telescopes, CO could be detected in distant comets (heliocentric distance* $r = 3$ *to 30 AU) brighter than* $m_1 = 13$ *to 14, provided that it is responsible for their observed activity.*

1. Introduction

One of the key problems in cometary astronomy is to connect the readily available total visual magnitudes (m_1) to physical quantities, for any given comet of interest. Indeed, for the purpose of planning professional observations, one would be interested in having the most recent quantitative information on each comet concerning its activity or mass-loss rate (dust- and gas- production rates). Previous work (*e.g.*, Jorda *et al.* 1992) has established relationships between the outgassing rate of water and the heliocentric magnitude ($H_\Delta = m_1 - 5 \log \Delta$, corrected for the comet-earth distance, Δ). My study now aims to extend this kind of correlation to heliocentric distances (r) beyond $r = 3$ AU. Farther than 3 AU from the sun, cometary activity is still frequently observed, but cold temperatures do not allow water sublimation to be efficient enough to sustain such activity. Since 1994, submillimeter observations have enabled us to measure the production rate of CO in several distant comets, which will be compared to the visual magnitudes measured at the same time.

2. CO in comets

The thorough chemical investigation of comets in recent decades (Bockelée-Morvan *et al.* 2000; Crovisier *et al.* 2000) has given us a better idea of the chemical content of the cometary ices.

Water is the main constituent, but other more volatile species like CO, CO_2, CH_3OH, CH_4, H_2CO, and H_2S have abundances relative to water that can reach or exceed 1%. Among them, CO (and to a lesser extent CO_2; Crovisier *et al.* 1999a, 1999b) can be considered as a major species with an abundance up to 25%. Being much more volatile than water, CO is likely the main gaseous species to escape from cometary nuclei far from the sun. Recent observations have, however, shown significant differences between comets, with CO/H_2O ratios in the coma close to the sun ranging from $< 2\%$ to $> 20\%$.

The long-term investigation of the activity of comet C/1995 O1 (Hale-Bopp), which is rather rich in CO ($\sim 20\%$ relative to water, near perihelion), has shown that the sublimation rate of CO is overtaking that of water beyond $r = 3$-4 AU (Biver *et al.* 1997, 1999a; Figure 1 of this paper). This was expected, as the sublimation of pure water ice is expected to be significant only when $r < 3$ AU, while CO could still sublimate in the coldest part of the inner solar system (up to $r = 50$ AU) — although the mechanism for releasing CO, likely partially trapped in water ice, is not fully understood.

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* Written as a detailed version of a paper presented at the IWCA II, Cambridge, England, 2001 Aug. 14-16.

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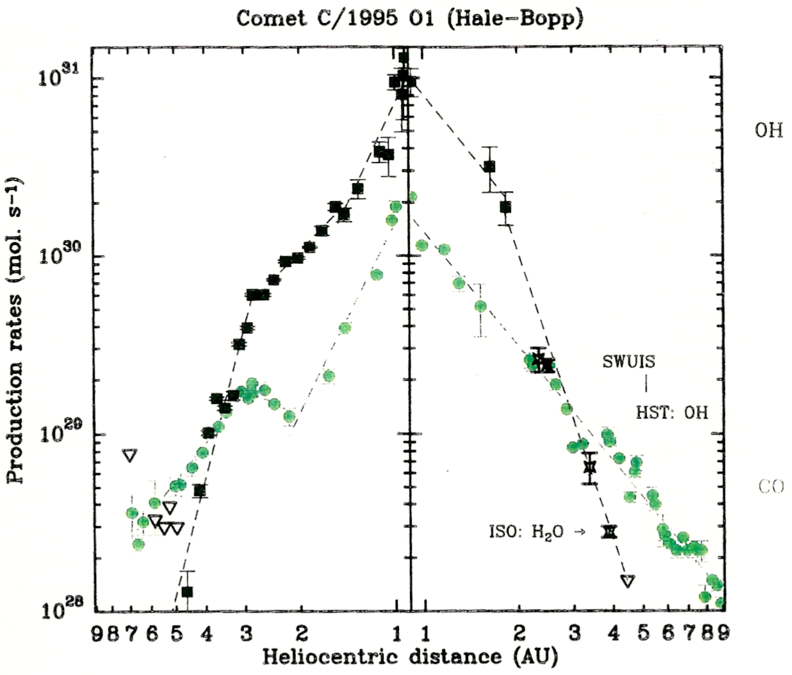

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Figure 1 (above). Evolution of the OH and CO production rates in comet C/1995 O1 (Hale-Bopp) versus time and heliocentric distance (left: pre-perihelion; right: post-perihelion). Circles represent CO, and squares represent OH, production rates from observations at Nançay (Biver et al. 1999a, 1999c). Crosses represent post-perihelion observations from space, as follows: Southwest Ultraviolet Imaging System (OH at 2.3 AU; Stern et al. 1999); Hubble Space Telescope (OH at 2.5 and 3.4 AU; Weaver et al. 1999); Infrared Space Observatory (H_2O at 3.9 and 4.6 AU; Crovisier et al. 1999b). Triangles are upper limits for OH or water-production rates (the water-production rates differs from those of OH by only 10%).

Figure 3 (below). Plot of the CO-production rates (or upper limits: downward-pointing triangles) versus heliocentric magnitudes for five comets observed at $r > 2.8$ AU.

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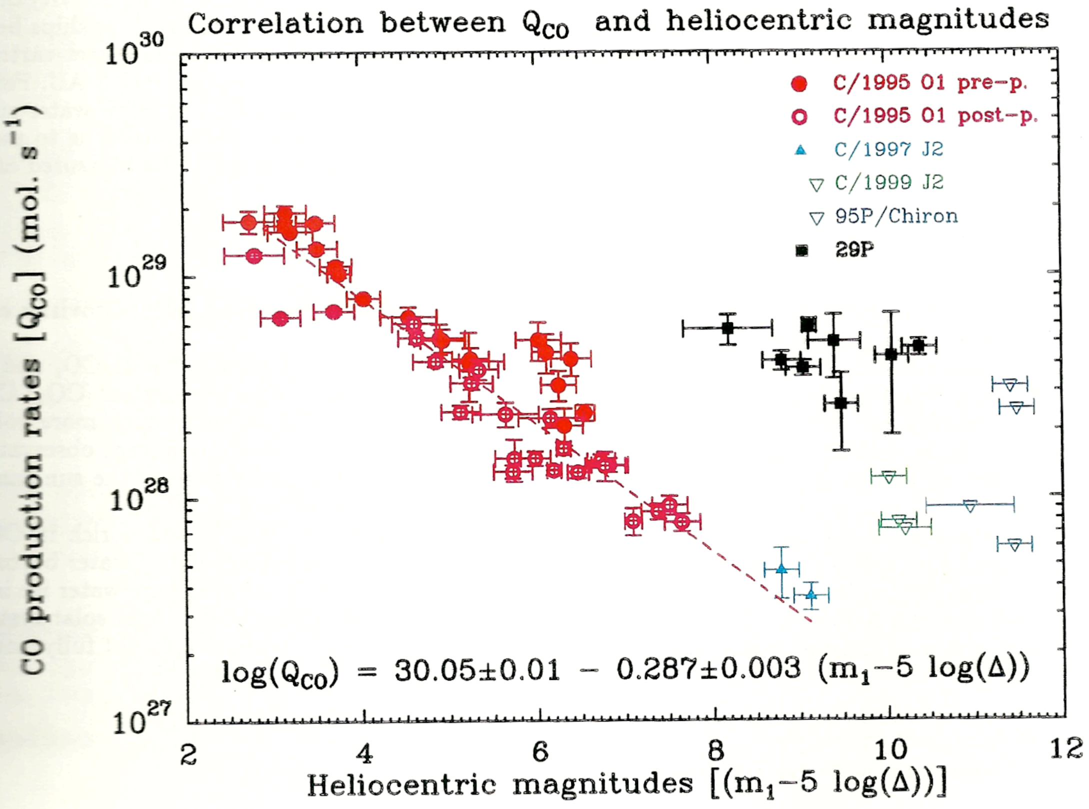

3. Production rates of water and CO

3.1. Spectroscopic observations

Most stable molecules like H_2O or CO are observable in the infrared to millimeter range (1μm to 3mm wavelength). A poly-atomic molecule can be in several discrete energy states following quantum theory. It stores energy in rotation and vibration of its atoms bindings and these energy levels are characterized by one or several integer quantum numbers. The energy step between two vibrational energy levels is about two orders of magnitude larger than between two rotational energy levels, and corresponds to the energy of infrared photons versus sub-millimeter to millimeter photons for rotational energy steps. What we observe are the photons emitted by molecules losing energy when they go down one step (in most cases) in energy level. The lowest energy level (of vibration or rotation) is called the ground level and corresponds to the minimum quantum number(s) (0 in many cases). Otherwise we talk about excited states and a molecule can be both in vibrational and rotational excited state. This makes the infrared spectrum usually more complex due to the large number of transitions possible, while sub-millimeter spectra only show transitions between rotational energy levels in the ground vibrational state. Also the simpler the molecule structure is (like CO), the simplest its energy diagram is (fewer quantum numbers needed to describe it).

Also to be in a given excited state, the molecule first needs to gain energy: either via the absorption of a photon (e.g. emitted by the Sun) or through collisions with other molecules: in a local equilibrium case the gas temperature will then characterize the distribution of the molecules on the various energy states.

There are several direct and indirect ways of measuring the water-production rate. Until recently, it has nearly been impossible to observe water vibrational (infrared) or rotational (submillimeter) transitions from the ground, because the earth's atmosphere contains water vapor and is opaque at these frequencies.

The Infrared Space Observatory (ISO) has been able to observe several of these lines in a few comets during its 30-month lifetime (Crovisier *et al.* 1999a, 1999b). Also recently, some specific vibrational lines of water have been observed in some bright comets in the infrared from a high-altitude dry site like Mauna Kea, at frequencies where atmospheric absorption is small (Dello Russo *et al.* 1997).

Most molecules are broken apart into radicals and atoms by the solar ultraviolet photons in a few hours at 1 AU. Water molecules photo-dissociate into OH + H in 90% of the cases, as a first step. The final photodissociation result is O + 2H. OH-radical lines have been observed for more than two decades either in the near-ultraviolet (*e.g.*, from space by the International Ultraviolet Explorer and Hubble Space Telescope) or at the radio wavelength of 18 cm. At such radio wavelengths, the atmosphere is totally transparent, and OH is observed with radio telescopes such as that at Nançay in France (*e.g.*, Bockelée-Morvan *et al.* 1990). Hydrogen atoms have been recently observed in detail in the ultraviolet at 121.6 nm (Lyman-α line) by the Solar Wind ANisotropies (SWAN) experiment aboard the SOHO spacecraft (Bertaux *et al.* 1998; Combi *et al.* 2000). The conversion of the production rates inferred for these "daughter" species OH and H into the outgassing rate of water is $Q_{\mathrm{H_2O}} = 1.1\ Q_{\mathrm{OH}} = 0.5\ Q_{\mathrm{H}}$.

Due to the presence of some CO in the earth's atmosphere, CO infrared lines can be observed from the ground only if the velocity of the comet relative to the earth is large enough. The telluric absorption lines are relatively narrow, and a "Doppler-shift" (displacement of the cometary lines, thanks to the Doppler effect) of a few tens of km/sec allows ground-based observations. In the millimeter to submillimeter range, the atmospheric transparency is generally better, although the shorter wavelengths require drier conditions, requiring high altitude (*e.g.*, Pico Veleta, Spain, at 2900 m, or Mauna Kea at 4100 m). In this wavelength range, we observe the transitions between rotational energy levels characterized by consecutive quantum number J, belonging to the ground vibrational state of the CO molecule. CO is also observed in the ultraviolet, but excitation processes are more complex and some of these CO lines are sensitive to the product of photo-dissociation of CO_2.

We will focus here on data from submillimeter observations of the CO transitions $J = 1 \rightarrow 0$ at 2.6 mm [later called CO(1-0)], $J = 2 \rightarrow 1$ at 1.3 mm [CO(2-1)], and $J = 3 \rightarrow 2$ at 0.87 mm [CO(3-2)]. Since the first detection of the CO(2-1) line in comet 29P at 6 AU from the sun in 1993 (Senay and Jewitt 1994), these lines have proven to be the best and easiest way to detect this molecule far from the sun. Indeed, cometary outer comae are usually very cold, especially at $r > 3$ AU (10-30 K) and submillimeter observations can sample "cold" gas. In contrast, other wavelengths (ultraviolet or infrared) require a significant amount of solar radiation (*i.e.*, being closer to the sun) to excite the observed transitions.

In addition, spectra at radio wavelengths are obtained with a very high spectral resolution, which are converted into Doppler velocities of the molecules relative to the cometary nucleus. They provide us with key information on gas dynamics: its expansion velocity and outgassing pattern. We often observe an asymmetry with a larger outgassing towards the sun, as can be deduced from most of the spectra like those in Figure 2. The simultaneous observation of several lines [*e.g.*, CO(1-0), CO(2-1), and CO(3-2)] — namely in comets C/1995 O1 and 29P (Biver *et al.* 1999a; Crovisier *et al.* 1995) — also yields essential information on the gas temperature.

3.2. Computation of outgassing rates

At radio wavelengths, a blackbody radiation spectrum follows the Rayleigh-Jeans approximation, which states that the energy radiated is proportional to its temperature. We use this approximation to express radio line intensities in degrees Kelvin (K). There are different "temperature" scales related to the exact beam pattern (equivalent to the diffraction pattern of an optical device) of the radio telescope. The "main-beam brightness temperature" scale is used here (Figure 2).

The conversion of the line-integrated intensity into the CO-outgassing rate, Q_{CO}, is done in several steps, summarized here:

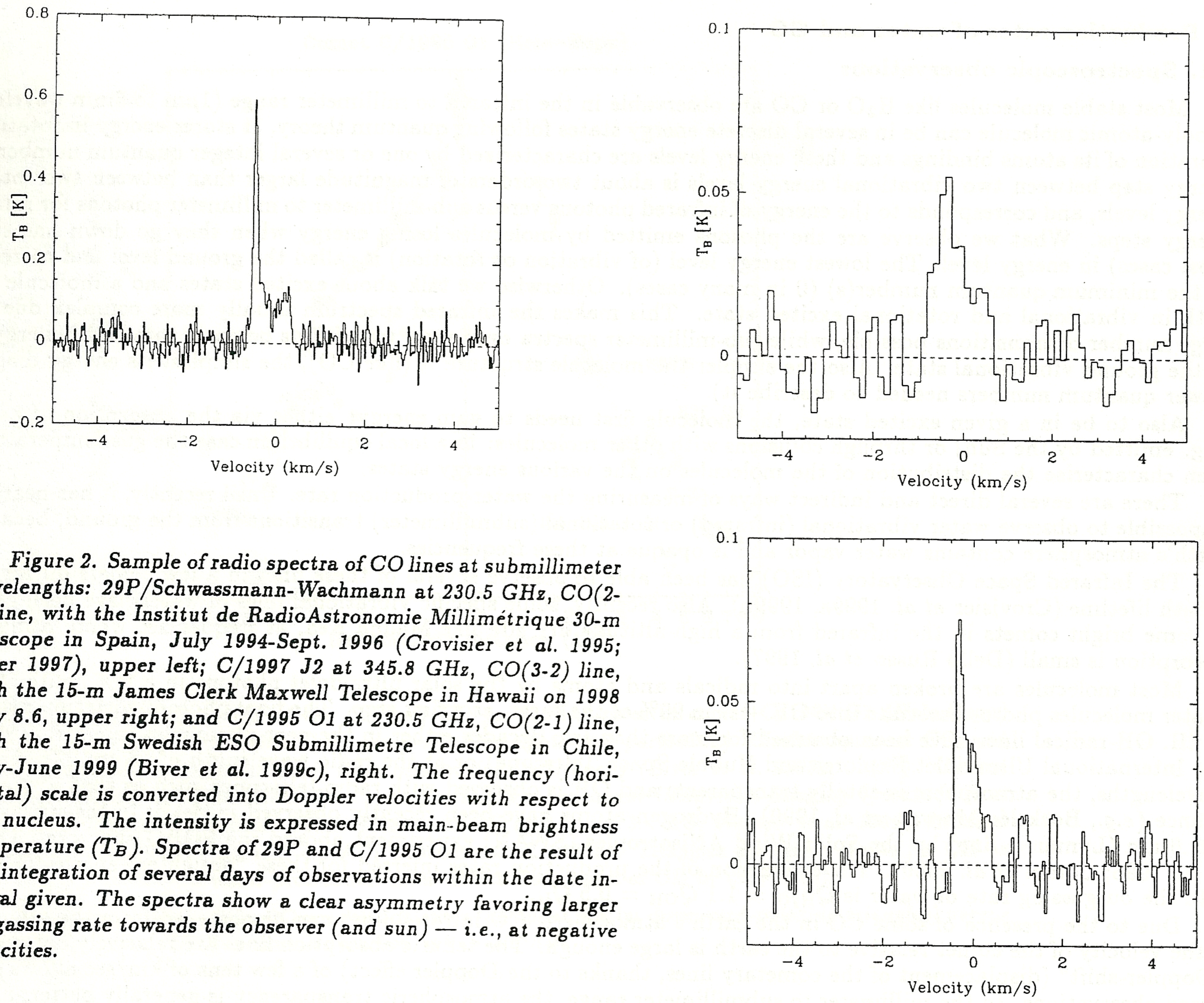

Figure 2. Sample of radio spectra of CO lines at submillimeter wavelengths: 29P/Schwassmann-Wachmann at 230.5 GHz, CO(2-1) line, with the Institut de RadioAstronomie Millimétrique 30-m telescope in Spain, July 1994-Sept. 1996 (Crovisier et al. 1995; Biver 1997), upper left; C/1997 J2 at 345.8 GHz, CO(3-2) line, with the 15-m James Clerk Maxwell Telescope in Hawaii on 1998 July 8.6, upper right; and C/1995 O1 at 230.5 GHz, CO(2-1) line, with the 15-m Swedish ESO Submillimetre Telescope in Chile, May-June 1999 (Biver et al. 1999c), right. The frequency (horizontal) scale is converted into Doppler velocities with respect to the nucleus. The intensity is expressed in main-beam brightness temperature (T_B). Spectra of 29P and C/1995 O1 are the result of the integration of several days of observations within the date interval given. The spectra show a clear asymmetry favoring larger outgassing rate towards the observer (and sun) — i.e., at negative velocities.

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- The use of the "Haser model" (Haser 1957), which describes the gas density, $n_{\rm CO}$, versus the distance, R, to the nucleus of the comet as:

$$n_{\rm CO}(R) = \frac{Q_{\rm CO}}{4\pi R^2 v_{\rm exp}} \exp\left(\frac{-R}{v_{exp}\tau_{\rm CO}}\right).$$

This formula assumes a stationary regime at a constant radial-expansion velocity, $v_{\rm exp}$, and CO expanding from the nucleus. The photodissociative lifetime of CO, $\tau_{\rm CO}$, is proportional to the square of the heliocentric distance (r^2) and is so long for this molecule (> 3 months at $r = 3$ AU) that it can nearly be considered as infinite.

- The determination of the relative population, $p_J(R)$, of the different energy levels (J) of the molecules throughout the coma; *i.e.*, the fraction of the molecules that are on the given energy states J, which uses the information on the gas temperature (see section 3.1). The energy radiated in the J–J-1 transition is proportional to p_J in the first approximation.
- The volume integral — of the density multiplied by the population profile, weighted (multiplied) by the beam pattern of the radio-telescope, $\eta(\rho)$ — is computed.[2] This angular sensitivity[3] is typically of circular gaussian shape with a full-width-at-half-maximum (FWHM) sensitivity $\theta_B = 10''$ to $1'$. Typically

$$\eta(\rho) = \frac{1}{2\pi\sigma_B^2} \exp\left(\frac{-\rho^2}{2\sigma_B^2}\right), \quad \text{with } \sigma_B = \frac{\Delta \tan(\theta_B)}{2\sqrt{2\ln(2)}}.$$

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[2] Note that the two-dimensional integral of $\eta(\rho)$ itself is equal to 1.0, so that this function 'beam pattern' can also be considered as a weighting function.

[3] $\rho = R\sin(\phi)$ being the projected radius on the sky, where ϕ (called "co-latitude" in spherical coordinates) is the angle subtending vectors **D** and **R**, with **D** on the z axis pointing towards the earth/observer and **R** being the vector comet → M, where M is a point at distance R from the comet's nucleus in the coma

And the volume integral to compute the line intensity is

$$\int\int\int n(R)p_J(R)\eta(\rho)dV = \int_{R=0}^{\infty}\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi} n(R)p_J(R)\eta[R\sin(\phi)]R^2\cos(\phi)d\theta d\phi dR$$

In this first description we also assume spherical symmetry of the coma. Further details can be found in Crovisier (1987) and Biver *et al.* (1999b), and the main points to note are:

- The modeling is fairly simple, and inferred CO-production rates are not very sensitive to the various parameters (v_{exp} and gas temperature, which are estimated directly from observed lines), within acceptable ranges or uncertainties.
- The CO lines are optically thin, which implies that Q_{CO} is proportional to the integrated line intensity, and modeling the lines shapes usually produces good fits to the observations.
- On the other hand, radio observations are like single-pixel "blind" observations: for observations of the CO(2-1) line, the gaussian beams have FWHM values of only $10''$ to $30''$, depending on the radio telescope. There are no "finder-scope" optical tracking options, making such observations very sensitive to ephemeris and pointing uncertainties (at best, around $2''$).

Besides the limit of the signal-to-noise ratio (S/N) of the observations themselves, I estimate that the uncertainty in the derivation of production rates of CO due to the modeling is below 15% — assuming that CO is a parent molecule (none is coming from an extended source in the coma; cf. DiSanti *et al.* 1999).

4. Correlation with visual magnitudes

Up to the year 2000, CO has been securely detected in three comets at $r > 3$ AU (29P, C/1995 O1, and C/1997 J2; Figure 2), and searched for in a few other comets. The lowest upper limit given for comet C/1999 J2 (Skiff) actually corresponds to a 3.5σ marginal signal, mostly coming from the June 1999 part of the integration, but not confirmed anywhere else or at any other opportunity. Comet C/1997 J2 was securely detected twice, in March and July 1998 at the James Clerk Maxwell Telescope (JCMT) on Mauna Kea at $r = 3.05$ and 3.27 AU, respectively. The observations of C/1995 O1 cover a 4-year-long monitoring, as presented at the "Asteroids, Comets, Meteors" meeting at Cornell University in July 1999 (Biver *et al.* 1999c; see also Biver *et al.* 1997 and 1999a). This monitoring is still going on, and data selected here cover the heliocentric ranges $r =$ 6.8-2.8 AU inbound and 2.8-9.2 AU outbound. Comet 29P has also been observed on several occasions between 1994 and 1999, between $r = 6.1$ and 6.3 AU. C/1999 J2 was observed at 7.3-7.2 AU between June 1999 and July 2000, and 95P/Chiron was observed between $r = 8.9$ and 9.5 AU in 1998-1999. Other searches for CO in 95P were done earlier, closer to its perihelion, in June-November 1995 ($r = 8.5$ AU). Unfortunately, most of these observations were close to solar conjunction, and no visible observations were done at the same time. If the comet did not vary much, its interpolated magnitude was around $H_\Delta = 10.8$, but upper limits ($Q_{CO} < 10^{28}$ molecules/sec; Rauer *et al.* 1997) do not yield better constraints. A marginal detection in June 1995 ($Q_{CO} \approx 2 \times 10^{28}$ molecules/sec) claimed by Womack and Stern (1999) was not further confirmed, and lower limits, as those presented here, have been obtained since.

Figure 3 (page 86) shows the CO-production rates deduced from the observations *versus* heliocentric magnitudes. Error bars on the production rates are the result of uncertainties on the line intensities. When no line is detected, the 3σ upper limit is converted into a production rate and represented by a downward-pointing triangle. The visual magnitudes (m_1) were extracted from the *ICQ* archives (issue numbers 91-107). We mainly used those of the most experienced observers (Green 1998) — correcting for, or excluding, those with large systematical biases. We selected m_1 data available during an interval of ± 2 to 3 days around the time of each CO observation. They were then averaged and corrected for the comet-earth distance — *i.e.*, converted into heliocentric magnitudes $H_\Delta = m_1 - 5 \log \Delta$. Their standard deviation is plotted in Figure 3 as horizontal error bars. Some radio observations could not be used, as there were no magnitude estimates available for the same period, the comet being generally too close to the sun in the sky.

We then applied a linear regression, fitting the points corresponding to the data sets (H_Δ, $\log Q_{CO}$) to a straight line. In the fitting process, we assigned a relative weight to each such data set. We investigated three different possibilities and obtained the following results:

- Same weight for all points (or 1 for all):

$$\log Q_{CO} = (29.98 \pm 0.08) - (0.268 \pm 0.015)H_\Delta$$

- Weighting according to the uncertainty on the H_Δ magnitudes; the weights are the inverse squares of the errors:

$$\log Q_{CO} = (30.26 \pm 0.04) - (0.322 \pm 0.007)H_\Delta$$

- Weighting according to the uncertainty on the production rate Q_{CO}:

$$\log Q_{CO} = (30.05 \pm 0.01) - (0.287 \pm 0.003)H_\Delta$$

In these linear regressions, we have taken into account C/1995 O1 and C/1997 J2 data. C/1997 J2 data insignificantly change the fitting from C/1995 O1 data points alone [$\log(Q_{CO}) = (30.06 \pm 0.01) - (0.288 \pm 0.003)H_\Delta$ in the third case]. The two points of C/1997 J2 do not yield a tight constraint: they fall in the continuation of C/1995 O1 observations, within error-bars, suggesting that the formulae might be extrapolated to low production rate and fainter comets. In Figure 3, we give the third formula, for which the linear correlation coefficient is $\rho = 0.93$; it would be 1.0 for points perfectly aligned, and 0.0 if no line could be fitted at all. The 29P data points are clearly offset by either 3.5 magnitudes or a factor 10 in Q_{CO} and cannot be fitted within the previous set of data.

In order to investigate the correlation between CO-production rates and visual magnitudes, one could have followed other steps. Another option could have been to use all magnitudes for a given comet and describe them by a smoothed formula such as $m_1 = H + 5 \log \Delta + 2.5n \log r$, eventually determining several (H, n) parameter sets to describe the brightness evolution of a comet during various time intervals (*e.g.*, C/1995 O1 in July 1995-June 1996, July 1996-Nov. 1996, Dec. 1996-Mar. 1997, and Apr. 1997-1999; see also Kidger *et al.* 1999). This may globally reduce the uncertainty but, on the other hand, smooths out any brightness surge. It is essential to correct m_1 magnitudes for the geocentric distance, but then we could also use $m_1 - 5 \log (\Delta r) = H_\Delta - 5 \log r$, which should reflect the true activity of the comet: the factor $5 \log (\Delta r)$ comes purely from the dependence of the brightness on the square of the distance. However, in this case, we find that the correlation factor ρ is worse for C/1995 O1 data alone ($\rho = 0.89$), and C/1997 J2 data points are even further away from the fitted line (including them yields $\rho = 0.85$, while we had $\rho = 0.93$ in the previous fit; cf. previous paragraph, above).

To take into account the outgassing of water molecules, one can make a weighted sum of the CO- and water-production rates, $Q_{\rm CO}$ and $Q_{\rm H_2O}$. Their crude sum represents the total gas-production rate in number of molecules, but the following sums have more physical sense:

- $m_{\rm CO}Q_{\rm CO} + m_{\rm H_2O}Q_{\rm H_2O}$ (*cf.* Table 1), which is the mass-production rate ($m_{\rm CO}$ and $m_{\rm H_2O}$ are respectively the molecular masses of CO and H_2O); or
- $\sqrt{m_{\rm CO}}Q_{\rm CO} + \sqrt{m_{\rm H_2O}}Q_{\rm H_2O}$, which is proportional to the total momentum of the CO + H_2O molecules, if they are decoupled, to which dust particles are more likely to be sensitive.

This is generally relevant to observations of CO-rich comets at $r < 3$ AU, especially when we want to merge the two correlation formulae relating $Q_{\rm CO}$, m_1 and $Q_{\rm H_2O}$, m_1 into a single description of the magnitudes based on production rates. (The two formulae would stay valid in the CO or H_2O dominated regimes.) Several of these possibilities were investigated with pre-perihelion observations of comet C/1995 O1 by DiFolco (1997), but they did not provide any large improvement over the simple search for correlation between $Q_{\rm CO}$ and H_Δ that has been done in my work.

In summary, $\log Q_{\rm CO} = (30.0 \pm 0.1) - (0.29 \pm 0.02)H_\Delta$. This formula is established on a fairly comprehensive data set for the two long-period comets that were detected: it covers two orders of magnitudes in production rate [$Q_{\rm CO} =$ (3-200) $\times 10^{27}$ molecules/sec)], a factor of 400 in brightness ($H_\Delta = 2.5$-9), and a factor of 3 in heliocentric distances (r = 3-9 AU).

Previous work on this correlation was done by DiFolco (1997) and by Bockelée-Morvan and Rickmann (1999), yielding a relatively similar formula,

$$\log Q_{\rm CO} = (30.00 \pm 0.04) - (0.256 \pm 0.009)H_\Delta,$$

which was only based on pre-perihelion data of comet C/1995 O1. Table 1 summarizes various correlation laws established between water- and/or CO-production rates and heliocentric magnitudes. We also give the correlation coefficient (ρ) when available, indicative of the quality of the fitting.

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Table 1. *Various correlation formulae between magnitudes and outgassing rates in comets.*

Fitted quantity "Q"	Correlation fit formula	ρ	Origin: Cometary data	Origin: Reference
$Q_{\rm H_2O}$	$\log(Q) = 32.4 - 0.4\ H_\Delta$		16 comets	Festou (1986)
$Q_{\rm H_2O}$	$\log(Q) = 30.6 - 0.25\ H_\Delta$		1P/Halley	Sekanina (1989)
$Q_{\rm H_2O}$	$\log(Q) = 30.74 - 0.24\ H_\Delta$	0.92	13 comets	Jorda et al. (1992)
$Q_{\rm H_2O}$	$\log(Q) = 30.49 - 0.24\ H_\Delta$		HB $3 > r_h > 1$ AU	BM & R (1999)
$Q_{\rm H_2O} + Q_{\rm CO}$	$\log(Q) = 30.78 - 0.33\ H_\Delta$	0.98	HB $7 > r > 1$ AU	DiFolco (1997)
$\mu_{\rm H_2O}Q_{\rm H_2O} + \mu_{\rm CO}Q_{\rm CO}$	$\log(Q) = 30.44 - 0.33\ H_\Delta$	0.98	HB $7 > r > 1$ AU	DiFolco (1997)
$Q_{\rm CO}$	$\log(Q) = 30.00 - 0.26\ H_\Delta$	0.96	HB $7 > r > 3$ AU	DiFolco (1997)
$Q_{\rm CO}$	$\log(Q) = 30.0 - 0.29\ H_\Delta$	0.94	HB $r \gtrsim 3$ AU and MD	*this paper*
$Q_{\rm CO}$	$\log(Q) \approx 30.2 - 0.2\ H_\Delta$	0.52	29P and C/1999 J2	*this paper*

Notes: $\mu_{\rm H_2O} = \frac{m_{\rm H_2O}}{m_{\rm H_2O}+m_{\rm CO}}$, $\mu_{\rm CO} = \frac{m_{\rm CO}}{m_{\rm H_2O}+m_{\rm CO}}$; ($m_{\rm CO}$ and $m_{\rm H_2O}$ are respectively the molecular masses of CO and H_2O).

HB = C/1995 O1 (Hale-Bopp); MD = C/1997 J2 (Meunier-Dupouy).

BM & R (1999) = Bockelée-Morvan and Rickmann (1999).

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5. Discussion

One should notice in Figure 3 that, when taking all data together, there is no good correlation ($\rho = 0.61$). The data from C/1995 O1 and C/1997 J2, which are the only well-detected long-period comets, yield a nice linear correlation between log $Q_{\rm CO}$ and H_Δ. Comet 29P is a peculiar object that could constitute another group of comets (to which comets like 95P or C/1999 J2 cannot yet be excluded). All comets above the dashed line on Figure 3 are either fainter visually or have a larger outgassing rate than expected from the correlation formula (*i.e.*, they have a much lower dust/gas ratio). Indeed, 29P is visually much fainter for a given CO-outgassing rate: the correlation formula based on C/1995 O1 and C/1997 J2 would put it at a visual magnitude between $m_1 = 8$ and 10 every time we observed it at radio wavelengths — which was never in fact the case. Data points are also relatively scattered for 29P reflecting a poor correlation between magnitudes (outbursts) and CO-production rates for this comet. Not many conclusions can be drawn from the observations of 95P and C/1999 J2, but one cannot exclude globally the correlation formula as providing a lower limit for the CO-production rate, given the brightness of the comet.

On the other hand, all investigations yield a similar slope in the correlation formula between log Q and H_Δ: -0.29 ± 0.03. This parameter is likely the more significant — not varying much from one comet to the other, in contrast to the other parameter of linear regression analysis (namely, the value of log Q for $H_\Delta = 0$ in the formulae), which may depend on the dust/gas ratio. The ultimate goal would be to find the scientific justification of this slope parameter. If the total brightness (or energy, E, radiated in the visible) was directly proportional to the total gaseous production rate ($E \propto 10^{-0.4H_\Delta}$, by definition), the slope would be -0.4. But the light reflected and scattered by the dust coma significantly contributes to visual magnitudes. Several factors that must be taken into account (in the conversion of the production rate into total brightness of the dust coma) depend on the heliocentric distance: illumination ($\propto r^{-2}$), dust velocity and coupling with the gas (whose expansion velocity is $\propto r^{-0.5}$; Biver *et al.* 1999a), and radiation pressure. On the other hand, if we assume an increase in the outgassing rate of CO as r^{-2} (it was actually slightly steeper for C/1995 O1), one would find $H_\Delta \propto 6.9 \log r$, implying a rather low activity index $n = 2.8$ for comets active at $r > 3$ AU.

As a consequence of these correlation formulae, on the basis of assumed evolution of gaseous production rates with heliocentric distances, one could derive activity indices:

- $Q_{\rm CO} \propto r^{-2}$ (which is proportional to the amount of energy received by the comet from the sun) implies $n = 2.8$, when CO is the dominant species.
- For water, at $r <$ 2-3 AU, $Q_{\rm H_2O} \propto r^{-2}$ implies $n = 3.3$ (based on the "$-0.24H_\Delta$" term in Table 1).
- But we have $Q_{\rm H_2O} \propto r^{-6}$ at $r > 3$ AU in the case of C/1995 O1 (Figure 1). Similar behavior could explain higher values for n ($Q_{\rm H_2O} \propto r^{-6}$ implies $n = 10$) for CO-depleted comets farther from the sun (*e.g.*, short-period comets).

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Table 2. *Using the correlation formula for observations of distant comets.*

Distance ($r \approx \Delta$)	$Q_{\rm CO}$ *inferred*		$<= m_1$ *observed*	Example
13 AU	0.3×10^{27} molec. s^{-1}	15 kg s^{-1}	18	C/1995 O1 in April 1993
9–13 AU	$0.8 - 1.5 \times 10^{27}$ molec. s^{-1}	35–70 kg s^{-1}	15–16	95P/Chiron
8.5 AU	0.1×10^{27} molec. s^{-1}	5 kg s^{-1}	18.5	C/1999 F1 (CATALINA)
7.9 AU	0.5×10^{27} molec. s^{-1}	25 kg s^{-1}	16	C/1999 S2 (McNaught-Watson)
4.2 AU	0.2×10^{27} molec. s^{-1}	10 kg s^{-1}	16	C/1999 S4 (LINEAR)
	Current 3-σ limit obtained in 5–6 days		$=> m_1$	
3 AU	1.2×10^{27} molec. s^{-1}	56 kg s^{-1}	12.8	
5 AU	2.0×10^{27} molec. s^{-1}	93 kg s^{-1}	13.1	
10 AU	4.5×10^{27} molec. s^{-1}	210 kg s^{-1}	13.4	
20 AU	9.0×10^{27} molec. s^{-1}	420 kg s^{-1}	13.8	
30 AU	14×10^{27} molec. s^{-1}	650 kg s^{-1}	14.0	

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Of key interest concerning a $Q_{\rm CO}$-H_Δ correlation formula is the obtaining of estimates of gas-production rates (for data analysis and planning purposes) from the visual-magnitude data provided by amateur observations. But in this case, we also rely on the significance of the other parameter of the regression analysis [the value of (log Q) for $H_\Delta = 0$]. As it is more likely to vary from comet to comet (likely higher, as we see for 29P), it also means that comets might be more CO-productive for a given magnitude than anticipated. Table 2 gives the 3σ limit [*i.e.*, the value of the radio signal below which any line detection cannot be secured ($S/N = 3$)], converted into CO-production rates, that can currently be achieved with radio telescopes. From the correlation formula (or lower limit on $Q_{\rm CO}$ that it would yield for a given H_Δ), if we translate this $Q_{\rm CO}$ into visual total magnitudes (assuming $r \approx \Delta$), it means that, in most comets brighter than $m_1 =$ 13-14 anywhere at $r > 3$ AU, a CO signal should be detectable — provided that their activity is governed by CO sublimation. In the near future, the Atacama Large Millimeter Array (ALMA) radio telescope should improve this detectability by nearly a factor 10, implying a gain of 3.5 magnitudes, thus putting many of the distant comets discovered every year within reach. Table 2 summarizes examples of comets recently observed and active far from the sun, and the application of the correlation formula to the current radio telescopes' performances.

6. Acknowledgments

Most of the CO observations of comet C/1995 O1 are the result of a worldwide effort to undertake a long-term monitoring. I am grateful to all the people involved in this large collaboration: D. Bockelée-Morvan (who also supported the work done on this subject by E. DiFolco, on which this paper is partially based), J. Crovisier, E. Gérard, P. Colom (France); A. Winnberg, H. Rickman, M. Gunnarsson (Sweden); F. Rantakyrö (Chile); D. C. Lis, J. K. Davies (USA); and many others that I may forget (Biver *et al.* 1999a, 1999c). I would also like to express special thanks to D. Bockelée-Morvan, J. Crovisier, and D. W. E. Green for their fruitful discussions to improve the paper.

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CCD Photometry of Comet C/1995 O1 (Hale-Bopp): 1995 - 2000*

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Abstract. Some 360 nights of CCD V-band photometry of the inner region (34″) of comet C/1995 O1 (Hale-Bopp) were carried out over an interval of 4.47 years, beginning when the comet was at heliocentric distance r = 7.1 AU, continuing in to r = 1.02 AU, and ending finally at r = 10.3 AU. Before perihelion, the magnitude brightened fairly steadily, with power-law exponent n = 2.55, and — as has been shown elsewhere (Liller 1997) — a well-established 20-day (± 5 days) periodicity persisted for a few months in late 1995. Following perihelion, the character of the light curve changed dramatically. The rate of fading was characterized at first by n = 4.46, but after passing r = 2.5 AU, the comet's brightness decreased irregularly at a rate of $n \approx$ 2.5, with the fading interrupted by at least five spectacular outbursts. The projected velocities of the outward flow of material ranged from 62 to 217 m/sec. After perihelion, the brightness of the quiescent comet averaged $\approx$ 0.18 magnitude fainter than before perihelion.

Introduction.

Shortly after the discovery of comet C/1995 O1 (Hale-Bopp), the author, having at his full-time disposal a fast Schmidt camera equipped with a CCD, embarked on a systematic program of broadband V photometry. The camera, a Celestron 0.2-m f/1.5 system designed for photography, had been provided to the author by NASA as a part of NASA's International Halley Watch (see Niedner and Liller 1987 for a description of the program and sample photographs). It has since been on indefinite loan to the author with the proviso that it be used in part for comet research, and is now installed in the author's private observatory in Viña del Mar, Chile. The CCD is an SBIG ST-5 camera having a TC-225 chip with 10-micron pixels; it is mounted at the "Newtonian focus" of the Schmidt camera. At the focus, a pixel measures 6″.8 × 6″.8; the field dimensions are 36′ east-west by 27′ north-south. In the light path is a minus-IR filter (Corion NR-400), chosen after much experimentation to produce a high-throughput passband with an effective wavelength close to that of the standard V system. Its long wavelength cut-off is at $\approx$ 7200 Å. Measurements of standard stars showed that, at a typical air mass of around 1.3 in the relatively humid atmosphere of the coastal location, the deviations from true V magnitudes over a wide range of $B-V$ colors were never more than a few hundredths of a magnitude (see Liller 1997 for a sample calibration curve).

Observing Program.

Observations were made on most reasonably clear nights when the author was at home, beginning 1995 August 3. This translates into occasional absences of a few weeks at a time, and otherwise an average of three times a week in the southern summer months, and once or twice a week in the winter, with somewhat poorer coverage when the comet was

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* **Written as a detailed version of a paper presented at the IWCA II, Cambridge, England, 2001 Aug. 14-16.**